Assignment 2

Define an appropriate language and formalize the negation of the following sentences using FOL formulas.

1.1a) Some people like Python.

FOL - Ǝx(People(x) ∧ Like(Python))

Negation - ∀x(¬People(x) ∨ ¬Like(Python))

1.1b) Every box contains at least one coin.

FOL – ∀x(Box(x) → Ǝy(Coin(y) ∧ Contains(x,y)))

Negation - Ǝx(Box(x) ∧ ∀y(¬Coin(y) ∨ ¬Contains(x,y)))

1.1c) All red objects are to the left of all green objects.

FOL – ∀x∀y(Red(x) Green(y) → Left(x,y))

Negation - ƎxƎy(Red(x) ∧ Green(y) ∧ ¬Left(x,y))

Define an appropriate language and formalize the following sentences using FOL formulas

1.2a) Every cat loves anyone who gives the cat a good food.

FOL - ∀xƎy(Cat(x) ∧ Person(y) ∧ Loves (x,y) → GivesFood(y,x))

1.2b) There are at least two rooms.

FOL – ƎxƎy(Room(x) ∧ Room(y) ∧ (x ≠ y))

2.1) Give a suitable structure, 𝒜 = (U𝒜, I𝒜), where U𝒜 = {1, 2, 3} such that under this structure 𝒜(F) = True. Justify

F = F1 ∧ F2 ∧ F3

F1 = ƎxƎyƎz((P(x,y) ∧ P(y,z) ∧ ¬(x = y) ∧ ¬(y = z)) → P(x,z))

F2 = ∀x∀y((P(x,y) ∧ P(y,x)) → x = y)

F3 = ∀x∀y(P(a,y) → P(x,b))

A suitable structure is x = 1, y = 1 and z = 1 and P(x,y) = T if x = y and a = x and b = y.

Convert into equivalent Skolem normal form. Justify

2.2a) ¬((∀x)P(x) → (∀x)(Ǝy)(Ǝz)Q(x,y,z))

≡ ¬(¬ ((∀x)P(x)) ∨ (∀x)(Ǝy)(Ǝz)Q(x,y,z))

≡ ¬(¬(∀x) ¬P(x) ∨ (∀x)(Ǝy)(Ǝz)Q(x,y,z))

≡ (∀x) P(x) ∧ ¬(∀x) ¬(Ǝy) ¬(Ǝz) ¬Q(x,y,z)

≡ (∀x) P(x) ∧ (Ǝx)(∀y)(∀z) ¬Q(x,y,z) – CNF

≡ (∀x) P(x) ∧ (Ǝw)(∀y)(∀z) ¬Q(w,y,z)

≡ (∀x)(Ǝw)(∀y)(∀z)(P(x) ∧ ¬Q(w,y,z)) – Prenex

≡ (∀x)(∀y)(∀z)(P(x) ∧ ¬Q(f(x),y,z)) - Skolem

2.2b) Ǝz(ƎxQ(x,z) ∨ ƎxP(x)) → ¬ (¬ƎxP(x) ∧ ∀xƎzQ(z,x))

≡ ¬Ǝz(¬(ƎxQ(x,z) ∨ ƎxP(x))) ∨ ¬ (¬ƎxP(x) ∧ ∀xƎzQ(z,x))

≡ ∀z(∀x¬Q(x,z) ∧ ∀x¬P(x)) ∨ ¬ (∀x¬P(x) ∧ ∀xƎzQ(z,x))

≡ ∀z(∀x¬Q(x,z) ∧ ∀x¬P(x)) ∨ (ƎxP(x) ∨ Ǝx∀z¬Q(z,x))

≡ ∀z∀x(¬Q(x,z) ∧ ¬P(x)) ∨ (ƎxP(x) ∨ Ǝx∀z¬Q(z,x))

≡ ∀z∀x(¬Q(x,z) ∨ ƎxP(x) ∨ Ǝx∀z¬Q(z,x)) ∧ (¬P(x) ∨ ƎxP(x) ∨ Ǝx∀z¬Q(z,x)) – CNF

≡ ∀z∀x(¬Q(x,z) ∨ ƎyP(y) ∨ Ǝy∀w¬Q(w,y)) ∧ (¬P(x) ∨ ƎyP(y) ∨ Ǝy∀w¬Q(w,y))

≡ ∀z∀xƎy∀w(¬Q(x,z) ∨ P(y) ∨ ¬Q(w,y)) ∧ (¬P(x) ∨ P(y) ∨ ¬Q(w,y)) – Prenex

≡ ∀z∀x∀w(¬Q(x,z) ∨ P(f(z,x)) ∨ ¬Q(w,f(z,x))) ∧ (¬P(x) ∨ P(f(z,x)) ∨ ¬Q(w,f(z,x))) - Skolem

Find the MGU if possible. Justify.

3a) {P(f(x,b),z), P(y,g(z))}

Not possible since cannot change z to g(z) since it is the same variable.

3b) {S(x,y,z), S(u,g(v,v),v)}

{S(u,y,z), S(u,g(v,v),v)}

{S(u,g(v,v),v), S(u,g(v,v),v)}

{S(u,g(v,v),v), S(u,g(v,v),v)}

MGU: [x/u, y/g(v,v), z/v]

3c) {Q(f(x),y,v), Q(z,g(w),h(z,y))}

{Q(f(x),y,v), Q(f(x),g(w),h(f(x),y))}

{Q(f(x),g(w),v), Q(f(x),g(w),h(f(x),g(w)))}

{Q(f(x),g(w),h(f(x),g(w))), Q(f(x),g(w),h(f(x),g(w)))}

MGU: [z/f(x), y/g(w), v/h(f(x),g(w))]

4.1) F = Ǝx(¬P(x) ∧ ¬P(f(v)) ∧ ƎzQ(z)) ∨ Ǝw(¬P(g(w,x)) ∧ ¬Q(x)) ∨ ƎyP(y)

¬F = ∀x(P(x) ∨ P(f(v)) ∨ ∀z¬Q(z)) ∧ ∀w(P(g(w,x)) ∨ Q(x)) ∧ ∀y¬P(y)

≡ ∀x∀z∀w∀y((P(x) ∨ P(f(v)) ∨ ¬Q(z)) ∧ (P(g(w,x)) ∨ Q(x)) ∧ ¬P(y))

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}} sub [y/g(w,x)] on clause 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}} use clause 2 and 4

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}} sub [x/z] 5

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}} use 1 and 6

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}} sub [y/x] on 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}} use 7 and 8

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}} sub [y/f(v)] on 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}, {¬P(f(v))}} use 9 and 10

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}, {¬P(f(v))}, {□}}

F is valid because S is unsatisfiable. S is unsatisfiable since the empty clause was found.

4.2) S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}} sub [y/x] on clause 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}} use clause 1 and 4

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}} sub [y/g(w,x)] on 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}} use 2 and 6

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}} sub [x/z] on 7

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}} use 5 and 8

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}} sub [y/f(a)] on 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}, { ¬R(f(a))}} use 9 and 10

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}, { ¬R(f(a))}, {□}}

S is unsatisfiable since the empty clause was found.